Generalized hypergeometric functions with several variables

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A restriction of the zonal spherical function of type A on a singular line is expressed by the generalized hypergeometric function (HG)

$$ {}_pF_{p-1}(\alpha_1,\dots,\alpha_p;\alpha'_1,\dots,\alpha'_{p-1};x)=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)}{(\alpha'_1)_n\cdots(\alpha'_{p-1})_n}\frac{x^n}{n!}. $$

Then the Gauss summation formula, or Harish-Chandra's c-function, of Heckman-Opdam's HG, which is a generalization of the zonal spherical function, is obtained by a connection formula of this generalized HG by [1].

We introduce the generalized HG of two variables

$$ \phi(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha_1)_m\cdots(\alpha_p)_m(\beta_1)\cdots(\beta_q)_n(\gamma_1)\cdots(\gamma_r)_{m+n}}{(\alpha'_1)_m\cdots(\alpha'_{p'})_m(\beta'_1)\cdots(\beta'_{q'})_n(\gamma'_1)_{m+n}\cdots(\gamma'_{r'})_{m+n}}\frac{x^my^n}{m!n!} $$

under the condition that the differential equation $\mathcal{M}$ satisfied by $\phi(x,y)$ has no irregular singularities, namely we assme

$$ p'-p+1=q'-q+1=r-r'. $$

We note that Appell's hypergeometric functions are examples.
The following problems will be discussed.

- Integral repsentaion of $\phi(x,y)$
- Rank and singularities of $\mathcal{M}$
- Construction of a base of local solutions at several singular points
- Connection fromula between these local solutions
- Necessay and sufficient condition for the irreducibility of $\mathcal{M}$
- Genelization of $\phi(x,y)$ to HG with more variables.

This work is in progress collabolated with S-J. Matsubara-Heo.

References

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